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A model-checking-based approach to risk analysis in supply chain consolidations

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Abstract. Supply chain strategy has become an important factor that dictates the successes of companies in today's competitive world. Nowadays more and more companies are tapping into the mergers and acquisitions in hope of getting the synergistic gain in supply chain consolidation. In this paper we use a model-checking-based approach to study the impact of different consolidation strategies on risks in supply chains and compare their capacity of risk reduction. We model stochastic behaviors of supply chains using an extension of Markov Decision Processes and translate the goal of risk analysis into a temporal logic. We then apply probabilistic model checking to analyzing risks inherent in a stochastic supply chain model. In our computational study, we consider three different consolidation strategies initially modeled in [18] and compare their capability of risk reduction in a generic three-echelon supply chain network. Our results reveal some key factors that improve the benefit of supply chain consolidation on risk reduction.

1. Introduction

Many companies embrace mergers and acquisitions as a growth strategy, which are widely believed to increase the net-value of the business [15]. When used with careful assessments and right strategies, mergers and acquisitions are considered as an effective way to improve the competitiveness of underlying corporations [2,13]. Among proposed benefits of mergers and acquisitions are increased market share, reduced overhead of production overlap, and increasingly important, improved supply chain operations.

In today's global economy, companies such as *Walmart* and *Costco* leverage their advantages on supply chains to reduce cost and increase profitability [4]. The result of such fierce competition is devastating to many smaller players in the market. One of examples of importance of supply chain in today's economy is *Kmart*'s fate in recent years. On January 22, 2002 *Kmart* filed for Chapter 11 Bankruptcy. Among contributing factors of such failure is the inefficiency of *Kmart*'s supply chain [10]. After emerging from the bankruptcy, *Kmart* announced to acquire *Sears, Roebuck and Company*, and one important factor behind *Kmart*'s decision was that the acquisition would improve supply chain operations of both companies and reduce cost. More recently, the goal of post-merger synergy for supply chain operations can be seen to be a motivation for a merger or acquisition activity.

In the pursuit of low cost and high efficiency, many companies engage themselves in the global supply chain expansion involving suppliers, distributors, retailers and logistics providers across multiple continents [6]. For example, *Kmart* sources their products from Asia, North America to its more than 2000 retail stores in the United States. After the successful merger with Sears which completed in 2005, *Kmart*'s parent company *the Sears Holding company* operates more than 3,800 full-line and specialty retail stores in the United States and Canada [3]. *Costco* operates its 544 warehouse stores in North America, South America, Asia, and Europe [5]. It sources merchandise from all over the world. Take banana, one of its signature products as an example. After the deal with Bonita, who was the solo provider of banana for many years, was broken

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up recently, *Costco* buys bananas from companies in five different countries [12]. With the operation in this kind of scale, companies' supply chains are more vulnerable than ever to risks that are naturally embedded in the system, such as machine failures, labor disputes, natural disasters, or even terrorist attacks. The question is, among different strategies of consolidating supply chains in mergers and acquisitions, which one brings more benefits to the constituent companies in terms of risk reduction, and under what circumstance?

Recently supply chain consolidation in mergers and acquisitions attracts much research interests. Most of existing research emphasize on the improvement of supply chain performance brought by consolidation. For example, Xu [18] studied synergistical gain of different consolidation strategies and identified some key factors affecting the success of supply chain consolidation. In this paper, we study how consolidation can impact risks in supply chains. Our research is motivated by the need from both industry and academia for better approaches to understand and manage risks in supply chain in a global context. Recent recalls on toys and pet food have served as a fresh reminder on risks in global supply chains. If left undressed, risks may derail entire supply chain operation.

To facilitate our study on risks in consolidated supply chains, we propose a novel computational framework for modeling and analyzing stochastic behavior of a supply chain. Our framework is based on probabilistic model checking [1], a formal verification technique developed in Computer Science for analyzing stochastic systems. Recent advances [9] in symbolic probabilistic model checking have drastically improved its efficiency and scalability. In past several years its application has been extended from computer-based systems to a wide range of other subjects such as biological pathway [8]. A typical supply chain contains a large number of stochastic elements in manufacturing and transportation processes. The scale and complexity of the problem make it an ideal candidate for probabilistic model checking.

Our approach is two-fold: first, we develop a formal framework to model the stochastic supply chain. The framework, Stochastic Merchandise Flow Model (SMF) is based on an extension of Markov Decision Processes. This framework provides a rigorous approach to modeling the dynamics of stochastic supply chain. SMF enables the composition of stochastic elements, for example, warehouses that may fail, and non-deterministic elements, for example, routing decisions, in a single framework. SMF also provides the foundation for the formal analysis of a stochastic supply chain; second, we establish a procedure for applying probabilistic model checking to stochastic supply chains. A probabilistic model checker such as PRISM [9] checks a stochastic system against a property encoded in a temporal logic. Use of a temporal logic allows us to express and check complicate stochastic properties not supported by existing domain-specific supply-chain risk analysis tools.

As an application of this new approach, we study different supply chain consolidation strategies and their impact on supply chain risk. Xu [18] modeled several popular supply chain consolidation strategies as mixed integer programming problems. She used these models to study the synergistical gain of supply chain consolidations in mergers and acquisitions. In this work, we extend her models to study the impact of different consolidation strategies on risks. Consolidations have been generally considered beneficial for reducing risks in supply chains. In this work we present quantitative measurement of risk reduction brought by these strategies. We also study key factors in supply chain consolidation that help reduce risks. The rest of the paper is organized as follows: Section 2 provides a brief introduction to probabilistic model checking and PRISM. Section 3 discusses SMF, a formal framework we introduced to model stochastic supply chains. Section 4 introduces the procedure we proposed for using probabilistic model checking to analyze stochastic supply chains. Section 5 discusses our computational study on different supply chain consolidation strategies and their impacts on risks. Finally Section 6 concludes the paper.

2. Probabilistic model checking with PRISM

Model checking is a formal verification technique that algorithmically checks a dynamic system against a temporal property encoded in some temporal logic. Probabilistic model checking extends classical model checking techniques with the ability of reasoning stochastic behaviors. In addition to simple "yes/no" answer, a probabilistic model checker also returns the probability with which a property may hold. Probabilistic model checking has been successfully used to study performance and reliability issues for a variety of subjects in Computer Science and other fields [11] where computational assessment of stochastic behaviors is the key to answers. A recent advance in probabilistic model checking is the development of sophisticated symbolic techniques that greatly improve the scalability and efficiency of decision procedures. We will discuss probabilistic model checking in context of the model checker PRISM.

Probabilistic model checking starts with the formal modeling of a stochastic system. Stochastic supply chains we are studying have probabilistic elements and nondeterministic elements. A classical method for modeling a stochastic system with nondeterministic behaviors is Markov Decision Process (c.f. [14]). Definition 1 defines EMDP, an extension of Markov Decision Process that we use as the mathematical foundation for our modeling framework.

Definition 1. (Extended Markov Decision Process) An Extended Markov Decision Process (EMDP) is a tuple $\langle \mathbf{v}, \mathbf{d}, \mathbf{V}^0, P, N \rangle$, where,

1. $\mathbf{v} = \langle v_1, \cdots, v_k \rangle$ is a vector of internal variables and its domain is $dom(\mathbf{v})$;
2. $\mathbf{d} = \langle d_1, \cdots, d_l \rangle$ is a vector of external variables and its domain is $dom(\mathbf{d})$;;
3. $\mathbf{V}^0 = \langle V_1^0, \cdots, V_k^0 \rangle$ is the initial valuation of $\mathbf{v}$.
4. $P : dom(\mathbf{v}) \times dom(\mathbf{d}) \rightarrow 2^{dom(\mathbf{v}) \times (0,1]}$ is the probabilistic transition function, such that for every $\mathbf{V}, \mathbf{D} \in dom(P)$, $\sum_{\langle \mathbf{V}', p \rangle \in P(\mathbf{V}, \mathbf{D})} p = 1$.
5. $N : dom(\mathbf{v}) \times dom(\mathbf{d}) \rightarrow 2^d om(\mathbf{v})$ is the nondeterministic transition function, such that $dom(N) \cap dom(P) = \emptyset$.

□

A state of an EMDP is defined by the valuation of its internal variables. The initial valuation $\mathbf{V}^0$ defines the initial state of the EMDP. External variables are the interface via which EMDP interacts with its environment. A probabilistic transition function defines outgoing probabilistic transitions from states. For example, $\langle \mathbf{V}, \mathbf{D} \rangle \rightarrow \{\langle w'_0, p_0 \rangle, \cdots, \langle w'_m, p_m \rangle\}$ represents that, if the current state is $\mathbf{V}$, and the valuation of the external variables $\mathbf{d}$ is $\mathbf{D}$, then, there is p_i of chance that the next state is w'_i, and $\sum_{i=0}^{m} p_m = 1$. In contrast, a non-deterministic transition function only specifies the set of the next states without probabilities. Note that non-deterministic transitions are *not* a special case of probabilistic transitions with probabilities distributed evenly. Instead, non-deterministic transitions represent a case in which EMDP may pick up the next state non-deterministically.

It shall be noted that EMDP is not more expressive than Markov Decision Process. Nevertheless, it does bring several benefits that make stochastic system modeling more effective: first, EMDP uses a vector of variables to encode the state space of a Markov Decision Process. This allows us to represent sets of states and transitions more efficiently. For example, one may use a predicate $(v_i > a) \wedge (v_j < b)$ to represent the set of all the states such that $v_i > a$ and $v_j < b$. Second, we make a clear distinction between probabilistic and nondeterministic behaviors by representing them separately using two different transition functions. In context of stochastic supply chains, probabilistic transitions represent risks in operations, for example, the probability of failure of a warehouse. Nondeterministic transitions represent choices in scheduling such as how to route merchandise flows. When an EMDP has only probabilistic transitions, it is reduced to Discrete Time Markov Process, in which system behaviors are left to the probability, as shown in Fig. 3. On the other hand, when an EMDP has only nondeterministic transitions, it is reduced to a nondeterministic finite automaton, as shown in Fig. 4.

$$
\begin{aligned}
f &::= A \mid \neg f \mid f \wedge f \mid \mathsf{P}_{\bowtie p} \phi \\
\phi &::= \mathsf{X} f \mid f \mathsf{U} f \mid f \mathsf{R} f
\end{aligned}
$$

where $A \in \mathcal{A}$ is an atomic proposition, $p \in [0,1]$, and $\bowtie \in \{\leq, \geq, <, >\}$.

Fig. 1. The syntax of PCTL [1,9].

We use the temporal logic PCTL [1] to express properties of a stochastic supply chain model. PCTL is a probabilistic extension of Computation Tree Logic (CTL). Figure 1 gives the syntax of PCTL in Backus-Naur form.

PCTL has two types of formulae: state formulae and path formulae. Semantically a state formula represents a set of states, and a path formula represents a set of paths. We let variables $\phi, \psi, \cdots$ and $f, g, \cdots$ range over path formulae and state formulae, respectively. PCTL is a propositional logic and it is built upon atomic propositions. An atomic proposition A represents a set of states by its semantic definition. The atomic propositions $\mathbf{T}$ and $\mathbf{F}$ stand for the set of all the states and the empty set, respectively.

PCTL uses a set of path operators *next* (X), *until* (U), and *release* R to express temporal patterns. A path formula $\mathsf{X} f$ holds on a path $s_1 s_2 \cdots$ if s_2 satisfies f. $f_2 \mathsf{R} f_1$ holds on a path ρ if f_1 holds for every state on ρ unless a state s_i satisfying f_2 "releases" such obligation, in which case f_1 does not have to hold for states after s_i. $f_1 \mathsf{U} f_2$ holds on a path ρ if f_1 holds for every state "until" a state s_i satisfying f_2, after which f_1 may or may not hold. Note that a subtlety is that f_2

eventually holds at some state on β in $f_1 \mathsf{U} f_2$ but not necessarily so in $f_2 \mathsf{R} f_1$. We also use two additional path operators *always* (G) and *eventually* (F). $\mathsf{G}f$ and $\mathsf{F}f$ stand for $\mathbf{F}\mathsf{R}f$ and $\mathbf{T}\mathsf{U}f$, respectively.

PCTL extends CTL with the probabilistic operator P, which attaches a probability to a path formula. For example, $\mathsf{P}_{>0.5}\phi$ is true for a state s if the probability that ϕ holds on the paths from s is greater than 0.5. PRISM also allows a user to query the probability associated with a path formula. Model checking $\mathsf{P}_{=?}\phi$ on a state s yields the probability that ϕ holds on the paths from s. PRISM also supports a bounded version of path operators. For example, $\mathsf{P}_{=?}(\mathsf{F}^{\leqslant 6} f)$ queries the probability that a state satisfying f can be reached within 6 steps from the current state. Interested readers may refer to [1] for a detailed discussion on the semantics of PCTL.

Since a Markov Decision Process may have nondeterministic elements, the probability associated with a path formula needs to be decided by checking all the possible resolutions of nondeterminism. PRISM supports two variants of P for model-checking Markov Decision Process: $\mathsf{P}_{\max}$ represents the best-case scenario in which a resolution of nondeterminism maximizes the probability that a path formula holds, and $\mathsf{P}_{\min}$ represents the worst-case scenario. Note that in our stochastic supply chain modeling, routing decisions are modeled as nondeterministic transitions. We use $\mathsf{P}_{\max}$ to force PRISM to search for the best routing strategy that improves the stochastic performance of a supply chain.

3. Modeling stochastic supply chains

The first step of model checking is to model a subject in a formalism that facilitates automated analysis. Before we define such a formalism for modeling a stochastic supply chain, we discuss the intuition behind it. To model the dynamics of a supply chain, we need to identify its states and transitions. A typical supply chain consists of suppliers, warehouses, retailers, and routes connecting them. Figurg 2 shows an example of a 2-echelon supply chain network. The label of an component indicates its failure and recovery rates. For example, the label for the warehouse w_1 indicates that there are 0.08 of chance w_1 may fail at any given time, and 0.8 of chance it recovers from failure. By default, an element without a label is always operational.

The purpose of a supply chain is to transfer goods. Its dynamics is characterized by merchandise flow in it. To identify the state of a supply chain, we need to consider the movement of merchandise and the status of each individual element of the supply chain. For instance, suppose that the supply chain in Fig. 2 carries two products, A and B. The state of the supply chain can be decided by A's and B's locations, and the status of each element, i.e., if an element is still operational. The stochastic model we are about to propose is the synchronized composition of element models and merchandise flow models, each of which is represented by an Extended Markov Decision Process.

Definition 2. (**Element EMDP**) An element EMDP (E-EMDP) is an EMDP $\langle\langle w\rangle, \langle W^0\rangle, P\rangle$, with no nondeterministic transitions, where,

1. The Boolean variable w indicates whether the element is operational.
2. The probabilistic transition function P is defined as follows: $P(\mathbf{T}) = \{\langle \mathbf{F}, p_{ed}\rangle, \langle \mathbf{T}, (1 - p_{ed})\rangle\}$ and $P(\mathbf{F}) = \{\langle \mathbf{T}, p_{de}\rangle, \langle \mathbf{F}, (1 - p_{de})\rangle\}$.
3. W^0 is the initial value of w.

We call w the status variable, p_{ed} operational probability, and p_{de} recovering probability. □

Intuitively, an element EMDP represents a two-state Markov Decision Process. An element may switch between operational state and failure state. Figure 3 shows the state space and transitions implied by the element EMDP for the warehouse w_1 in Fig. 2.

Definition 3. (**Merchandise EMDP**) Let $\mathcal{S}$ be a stochastic supply chain with k elements, and let w_i be the status variable of the i-th element. A merchandise EMDP (M-EMDP) for a product carried by $\mathcal{S}$ is an EMDP $\langle\langle v\rangle, \langle w_1 \cdots w_k\rangle, \langle f^0\rangle, N\rangle$, with no probabilistic transitions, where,

1. The domain of the variable v is the set of facilities in $\mathcal{S}$. A facility can be one of the following elements: a supplier, a warehouse, or a retailer.
2. An external variable w_i represents whether the i-th element is operational.
3. The nondeterministic transition function N is defined as follows: for every facility f and every route ff' emanating from it, $N(\langle f\rangle, \langle w_1 \cdots w_k\rangle)$ is,
   - $\{\langle f\rangle, \langle f'\rangle\}$, if $w_{e_f} = w_{e'_f} = w_{e_{ff'}} = \mathbf{T}$, or;
   - $\{\langle f\rangle\}$, otherwise.
4. f^0 is the initial value of v.

We refer to v as the location variable of the product. □

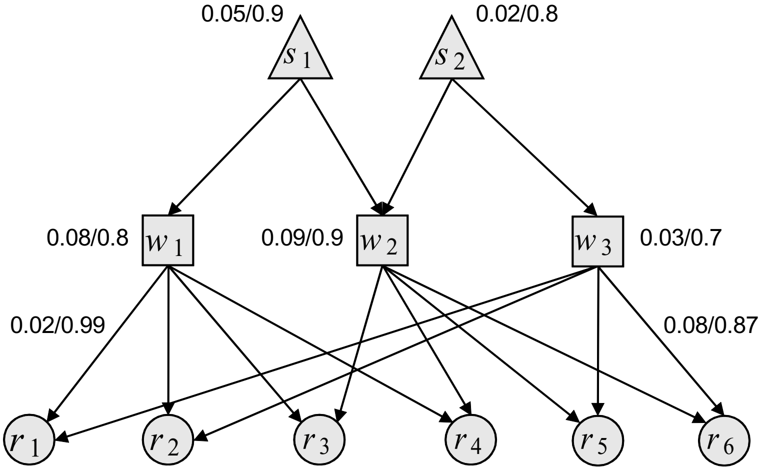


Fig. 2. A 2-echelon stochastic supply chain $\mathcal{S}_e$.

A merchandise EMDP represents how a product is transported through a stochastic supply chain. The location of the product is denoted by its location variable v. v's initial value f_0 represents the manufacturing facility for the product. The nondeterministic transition function specifies how the next location is chosen: the product can either stay at its current location f, or in case that f, a location f', and a route ff' are all operational, the product may also be transferred to f'. Unlike the probabilistic transition function in an E-EMDP, the transition function of an M-EMDP is nondeterministic: the model does not specify in what probability the next location is chosen from a list of eligible locations. A probabilistic model checker has to consider all the possible ways of resolving nondeterminism. In Section 4 we take advantage of such capability and force a probabilistic model checker to search for the best scenario for resolving nondeterminism. The answer produced by the model checker implies the optimal strategy for scheduling merchandise flow.

Figure 4 shows the state space and transitions implied by the M-EMDP for a product manufactured at the facility s_1. Note that it has a similar structure as the subset of its underlying supply chain in Fig. 2. This is because M-EMDP depicts the flow of a product in a supply chain and it shall have a similar structure as part of the supply chain the product may go through.

Definition 4. (**Stochastic Merchandise Flow Model**) Let $\mathcal{S}$ be a stochastic supply chain and $\mathcal{P}$ a set of products transported in $\mathcal{S}$, the stochastic merchandise flow (SMF) model of $\mathcal{S}$ is a synchronized parallel composition of EMDPs in $\mathcal{E} \cup \mathcal{M}$, where $\mathcal{E}$ is the set of all the E-EMDPs for elements of $\mathcal{S}$ and $\mathcal{M}$ is the set of all the M-EMDPs for products in $\mathcal{P}$. □

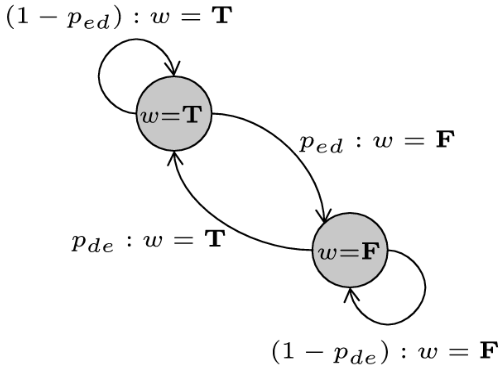


Fig. 3. The state space and transitions of the E-EMDP for the warehouse w_1.

We use Stochastic Merchandise Flow Model in Definition 4 to model the dynamics of a stochastic supply chain. A SMF model is a synchronized parallel composition of E-EMDPs and M-EMDPs. We write $N_{\mathcal{S}} = ||_{n \in \mathcal{E} \cup \mathcal{M}} n$ for the SMF model of a stochastic supply chain $\mathcal{S}$, where $\mathcal{E}$ and $\mathcal{M}$ are the set of E-EMDPs and the set of M-EMDPs in $\mathcal{S}$, respectively. States of a SMF model are identified by the valuation of status variables in E and location variables in M. Transitions are synchronized compositions of transitions of the E-EMDPs and the M-EMDPs. That is, $\langle w_1, \cdots, w_i, v_1, \cdots, v_j \rangle \rightarrow \langle w'_1, \cdots, w'_i, v'_1, \cdots, v'_k \rangle$ is a transition of $N_{\mathcal{S}}$ if and only if for every l such that $1 \leqslant l \leqslant i$, $w_l \rightarrow w'_l$ is a transition of $e_l \in \mathcal{E}$ and for every k such that $1 \leqslant k \leqslant j$, $v_k \rightarrow v'_k$ is a transition of $m_k \in \mathcal{M}$. Intuitively, a transition of $N_{\mathcal{S}}$ represents a discrete step in supply chain operations. During the discrete step, an E-EMDP may flip its status variable with a given probability, and an M-EMDP may nonde-

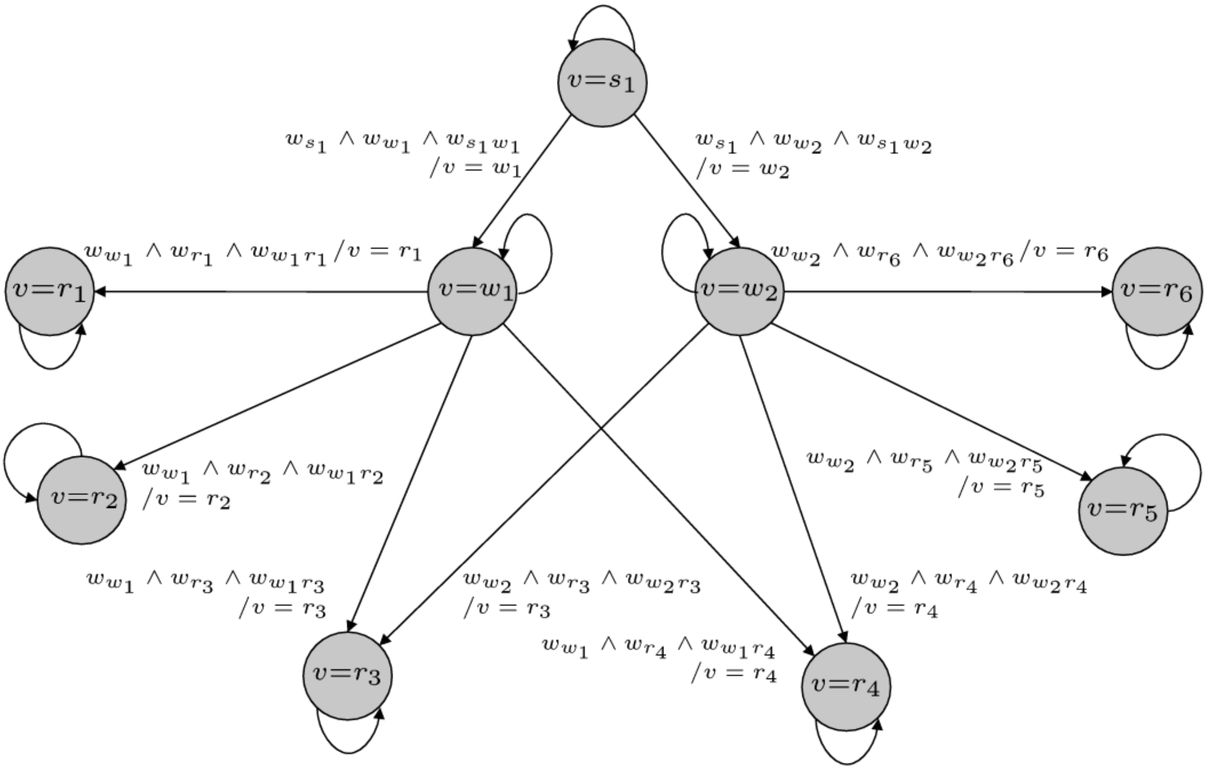


Fig. 4. The state space and transitions of the M-EMDP model for a product A manufactured in s_1.

terministically decide what will be the next location of the product it represents.

4. Analyzing stochastic supply chains

We use probabilistic model checking to automate the analysis of stochastic supply chain models. As we discussed in Section 3, the underlying formalism for stochastic supply chain models is EMDP, an extension of Markov Decision Process. Traditionally decision procedures for Markov Decision Process use dynamic programming technique and they are usually customized for targeted problem domains. Our choice of using probabilistic model checking as underlying analysis technology brings us several benefits: first, recent advances in probabilistic model checking provide efficient symbolic decision procedures. Symbolic probabilistic model checkers such as PRISM [9] use sophisticated symbolic techniques including Multi-Terminal Binary Decision Diagrams (MTBDD) [17]. By using probabilistic model checking, we are able to leverage the benefits of efficient decision procedures developed for verifying large-scale computer-based systems. Second, traditionally decision procedures analyze Markov Decision Processes by attaching to each transitions or states a *reward* and then optimizing a reward-based cost function. Although the reward-based approach is appealing, it also has its limitations. In comparison, a probabilistic model checker provides a generic decision procedure which not only supports the reward-based analysis approach but also enables us to specify sophisticated stochastic and temporal properties in a temporal logic. For example, the probabilistic model checker PRISM uses the probabilistic Computational Tree Logic (PCTL), and we can specify in PCTL a probabilistic temporary property P_{reach}: *what is the possibility that a product A can be delivered to a retailer r_i within 4 days after arriving a warehouse w_k.*

As part of analysis activities, we need to specify stochastic properties in a temporal logic, in our case, in PCTL. The basic building blocks of a PCTL formula are atomic propositions. Semantically an atomic proposition refers to a set of states in which by definition the proposition holds. We use an atomic proposition to label a set of states of special interest in analysis. Since a state is presented by a valuation of variables in EMDP, we may define an atomic proposition using a predicate over variables. For instance, to specify states of a product A "*arriving a warehouse* w_k", we define an atomic proposition $\mathcal{A}_{w_k} = (v_A = w_k)$, where v_A is A's location variable. Note that the predicate $v_A = w_k$ only constrains v_A's value, therefore, $\mathcal{A}_{w_k}$ specifies a set of states. That is, $\mathcal{A}_{w_k}$ holds at any state in which A is in the warehouse w_k.

The probabilistic model checker PRISM supports an extension of PCTL using bounded path formulae, which allows us to specify the number of discrete steps for which a property shall hold. As part of modeling and analysis tasks, one needs to decide the semantics of a discrete step in a SMF model based on the planning horizon of supply chain operations. For instance, if operations such as shipment are scheduled in term of days, a discrete step can be one day.

PCTL supports a set of temporal operators such as U (until), R (release), X (next), and their bounded version. Using PCTL, one may also encode the property such as in what probability a property holds on a system. PRISM supports the use of "?" in place of a real number to query in what probability a property holds. For example, in PRISM the PCTL formula for the property P_{reach} is $P_{\text{reach}} = \mathsf{P}_{\max\geqslant?}(\mathsf{G}((v_A = w_k) \rightarrow \mathsf{F}^{\leqslant 4}(v_A = r_i)))$. After checking P_{reach} on a stochastic supply chain model, PRISM returns the probability that the property holds for the model.

4.1. *Complexity of the probabilistic-model-checking-based risk analysis*

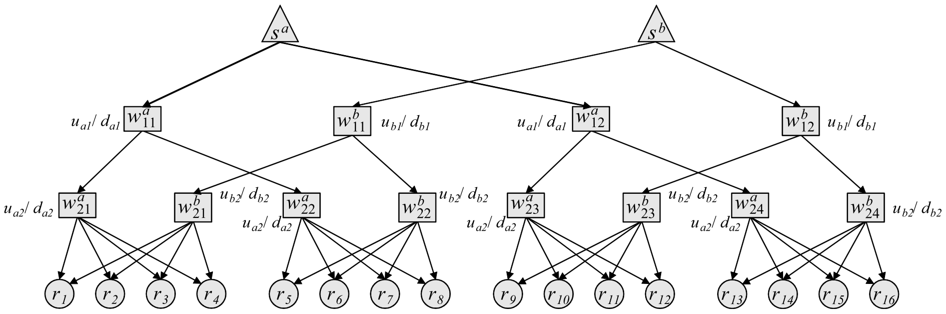
The computational cost of our approach comes from two sources: the cost of translating a supply-chain model to an EMDP, and the cost of model checking. Let $\mathcal{S}$ be a supply chain with S facilities and T routes. We define $|\mathcal{S}| = |S| + |T|$ as the size of $\mathcal{S}$. Let ϕ be the PCTL property we want to check. The translation procedure translates each facility to an E-EMDP, which has just two states. By Definition 3, the M-EMDP for $\mathcal{S}$ has at most S states and $T + S$ non-deterministic transitions, so it can be produced in $O(|\mathcal{S}|)$ time. Overall the translation takes $O(|\mathcal{S}|)$ and produces an EMDP of $O(|\mathcal{S}|)$ in size. The complexity of probabilistic model checking is linear in the size of ϕ and polynomial in the size of the EMDP [7]. Therefore the complexity of our approach is linear in the size of ϕ and polynomial in $|\mathcal{S}|$.

5. A computational study for risk analysis of supply chain consolidation strategies

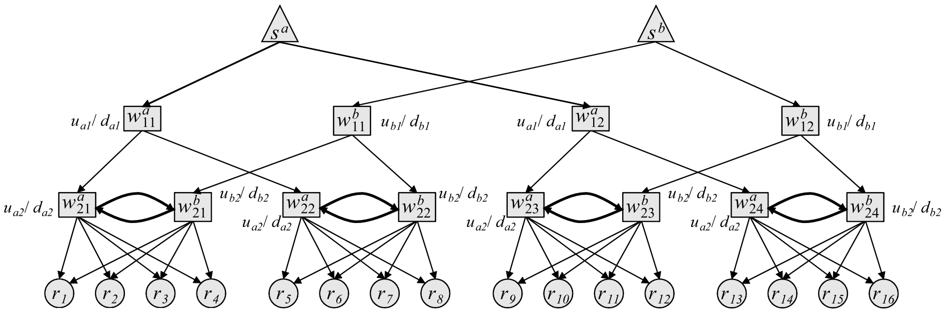
We apply the computational framework in Section 4 to studying risks in supply chain consolidations. In our computational study, we model different consolidation strategies and compare their benefits on risk reductions in different settings. The purpose of this study is to identify key factors in consolidations that may help reduce risks in a consolidated supply chain. Since our approach is based on model checking technique, which rigorously deduct mathematical proofs in an automated way, for a given supply chain model we can precisely evaluate its stochastic behaviors. Such precision gives the results of our computational study a high degree of confidence. In the case study we also extend underlying supply chain structures from two-echelon [16] networks to a more generic version of three-echelon networks.

5.1. *Modeling supply chains*

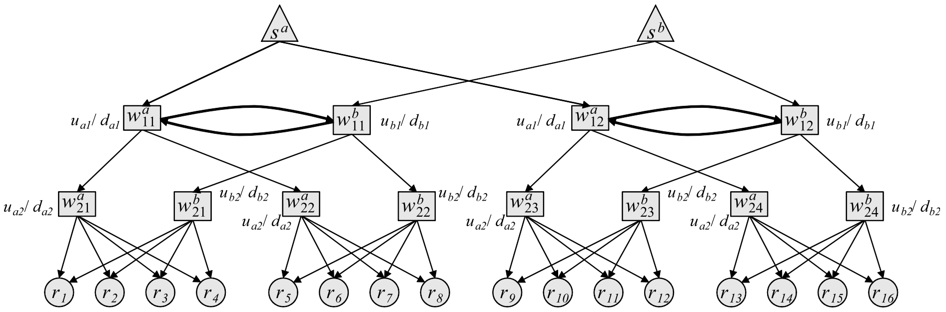
In this computational study we consider a general version of three-echelon supply chain. It extends two-echelon models in [16] with one more layer of warehouses. Figure 5(a) shows the configuration of the supply chain before a merger. It contains two separate networks operated by two independent companies. These companies are identified by their products a and b, respectively. Before the merger, two companies maintain two separate supply chain networks that serve the same geographical regions. Each company has its own supplier and operates its own warehouses. Each supply chain network has three echelons: one layer of suppliers, two layers of warehouses, and one layer of retailers. We denote w^k_{ij} for the j^{th} warehouse on the level-i operated by company $k \in \{a, b\}$. The model defines the partition of markets by their geographical and demographic characteristics. Retailers serving the same population base and/or the same region will be grouped as a market. In Fig. 5(a) there are four markets: $r_1 \cdots r_4$, $r_5 \cdots r_8$, $r_9 \cdots r_{12}$, and $r_{13} \cdots r_{16}$. Each company maintains its own subnetwork for each market. For instance, the market of $r_1 \cdots r_4$ gets its delivery from second-tier warehouse w^a_{21} for product a and from second-tier warehouse w^b_{21} for product b. These second-tier warehouses are in turn served by the warehouses on the first tier. This two-level structure of warehouses reflects the common practice in a global supply chain: level-1 warehouses represent hubs locat-



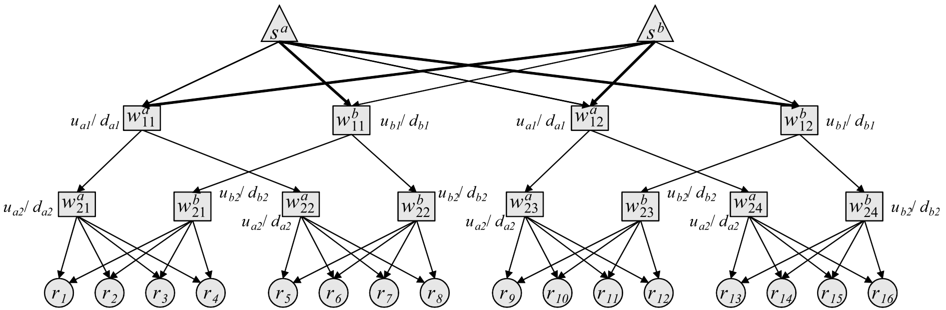
(a) $\mathcal{S}(d_{a1}, u_{a1}, d_{b1}, u_{b1}, d_{a2}, u_{a2}, d_{b2}, u_{b2})$: a supply chain with two networks operated separately by A and B



(b) $\mathcal{S}_{cd2}(d_{a1}, u_{a1}, d_{b1}, u_{b1}, d_{a2}, u_{a2}, d_{b2}, u_{b2})$: a consolidated supply chain with cross shipment among level-2 warehouses.



(c) $\mathcal{S}_{cd1}(d_{a1}, u_{a1}, d_{b1}, u_{b1}, d_{a2}, u_{a2}, d_{b2}, u_{b2})$: a consolidated supply chain with cross shipment on level-1 warehouses.



(d) $\mathcal{S}_{pp}(d_{a1}, u_{a1}, d_{b1}, u_{b1}, d_{a2}, u_{a2}, d_{b2}, u_{b2})$: a consolidated supply chain with product pooling.

Fig. 5. Supply chain configurations before and after a merger. d_{qi} and u_{qi} are the failure and recovery rates of a level-i warehouse operated by $q \in \{a, b\}$.

ed at ports, and level-2 warehouses represent distribution centers serving retailer markets.

To characterize risks, models are parameterized by failure rates and recovery rates of warehouses. We assume that the warehouses on the same level and operated by the same company share the same failure rate and recovery rate. For instance, we denote $\mathcal{S}(d_{a1}, u_{a1}, d_{b1}, u_{b1}, d_{a2}, u_{a2}, d_{b2}, u_{b2})$ for the supply chain in Fig. 5(a), where d_{qi} and u_{qi} are the failure and recovery rates of a level-i warehouse operated by

$q \in \{a, b\}$.

5.2. Modeling supply chain consolidation strategies

The benefits of consolidating supply chains have been a major contributing factor in recent mergers and acquisitions [4]. When two companies merge, their supply chains are consolidated. In [18] Xu has studied economical gains produced by supply chain consolidations. To identify key factors that affect economical benefits of supply chain consolidation, she modeled different consolidation strategies and did a comparison study on their performance on economical gain. In this work we are interested in the implication of consolidation strategies on risks in a consolidated supply chain. We extend Xu's work on modeling consolidation strategies and apply our analysis techniques to risk analysis of different consolidation strategies. We consider the following three consolidation strategies:

Level-2 cross-warehouse strategy. Under this consolidation strategy, the integrated supply chain allows cross shipments among level-2 warehouses previously operated by different companies. This strategy is useful when two companies' level-2 warehouses in the same market are geographically close to each other and hence cross-shipments among level-2 warehouses are a cost-efficient approach to integrate two supply chains and share resources. Figure 5(b) shows the integrated supply chain with level-2 cross warehouse strategy.

Level-1 cross-warehouse strategy. Under this strategy, the integrated supply chain allows cross shipments among level-1 warehouses previously operated by different companies. This strategy is useful when two companies' level-1 warehouses located close to each other at the same port and hence shipments among level-1 warehouses are a cost-efficient approach to share these warehouses. Figure 5(c) shows the integrated supply chain with level-1 cross warehouse strategy.

Product pooling strategy. Under this strategy, we allow shipments from a plant to a warehouse previously operated by a different company. Figure 5(d) shows the consolidated supply chain with product pooling strategy.

5.3. Modeling objectives of risk analysis

As part of the modeling efforts in our study, we model objectives of risk analysis in temporal logic PCTL. Since the purpose of a supply chain is to move merchandise from suppliers to customers, we measure risks in terms of probability of on-time delivery. Specifically, we consider the following representative property: *what is the probability products a and b, shipped from suppliers s^a and s^b, can be delivered to r_2 and r_2 within n steps using optimal routing strategy*. The property can be expressed as follows in PCTL:

$$g(n) = \mathsf{P}_{\max=?}\mathsf{G}[v_a = s^a \wedge v_b = s^b \rightarrow \mathsf{F}^{\leqslant n}(v_a = r_1) \wedge (v_b = r_2)]$$

Routing algorithms may drastically affect the performance of supply chains. A poor routing algorithm, or in other words, an inefficient operation it models, may offset the benefit of supply chain consolidation. To have a fair comparison among different consolidation strategies, we consider only situations with optimal routing decisions on risk reduction. Another reason for such assumption is that sub-optimal routing decisions reflect inefficiency of a supply chain operation and a company shall always strive to remove such inefficiency whether before or after a merger. Since our stochastic model in Section 3 has already integrated routing decisions as nondeterministic transitions to probabilistic behaviors of a supply chain, by querying what is the maximal probability $\mathsf{P}_{\max=?}$ we ask a model checker to explore only those executions representing optimal routing decisions. The result of model checking indicates the probability of on-time delivery with optimal routing decisions on risk reduction.

5.4. Experimental results and discussions

We design a set of experiments to compare the benefits of different consolidation strategies on risk reduction and identify key factors that affect their performance. All the experiments are carried out on a Windows 2003 R2 server with a 3.0 GHz Intel Xeon and 2 GB memory. We use PRISM version 3.2 as the underlying probabilistic model checker.

Our first experiment is on a supply chain with two separate networks, as shown in Fig. 5(a). This model assembles the before-merger scenario in which two companies run their own networks. The supply chain model is parameterized by failure and recovery rates shared by both networks. We denote $\mathcal{S}_s(u, d) = \mathcal{S}(u, d, u, d, u, d, u, d)$ for the supply chain parameterized with failure rate d and recovery rate u. The probability of on-time delivery within 4 steps is the result of model checking $\mathcal{S}_s(u, d) \models g(4)$. Figure 6 shows the experimental result. Since the failure rate is a major contributing factor to risks in the supply

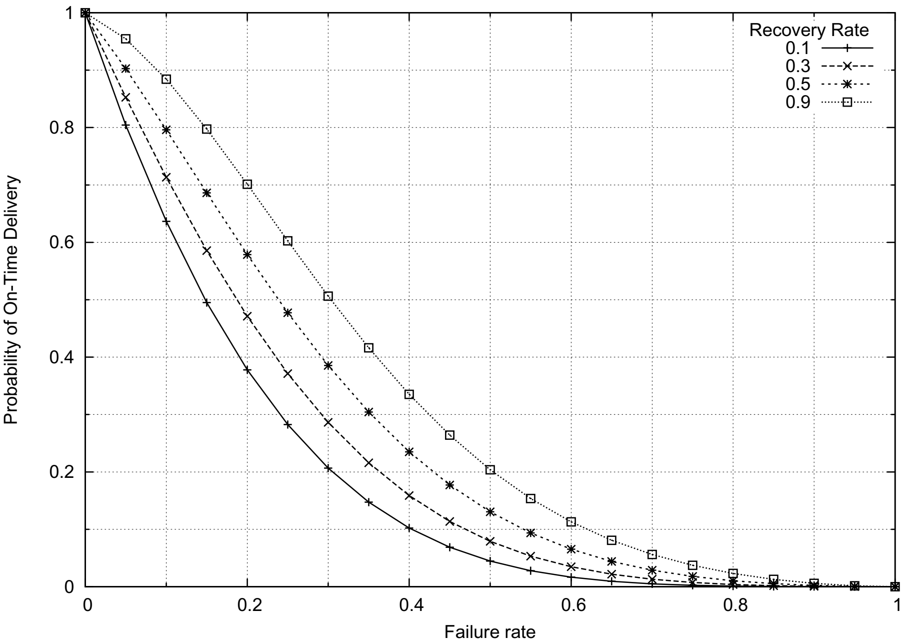


Fig. 6. Failure/recovery rates of warehouses and their impact on the probability of product on-time delivery before supply chain consolidation.

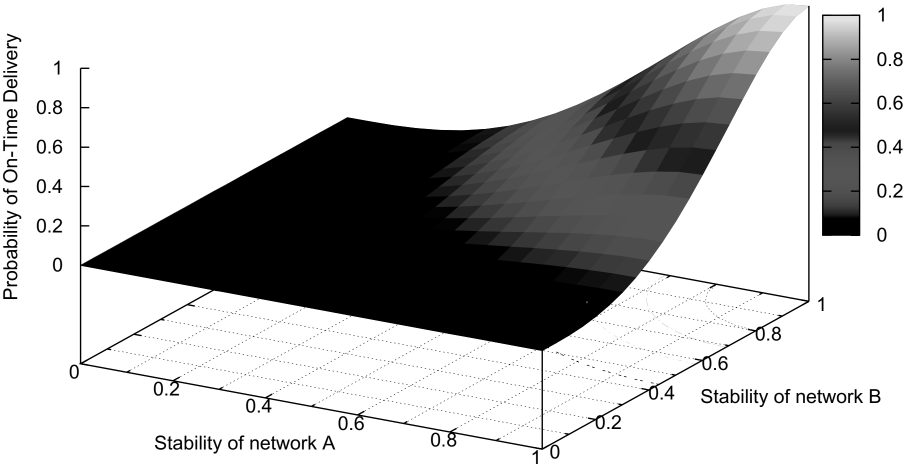


Fig. 7. Stability of networks and their impact on the probability of product on-time delivery before supply chain consolidation.

chain in Fig. 5(a), the probability of on-time delivery decreases when warehouses fail more frequently. A higher recovery rate may help reduce the risk, but its benefit is largely subjected to the failure rate, as indicated by the converge of lines towards the total failure of warehouses.

Figure 7 measures the stability of networks and their impact on risks in before-merge scenario. We parameterize the supply chain in Fig. 5(a) with stability coefficients p_a and p_b, and denote $\mathcal{S}^{(2)}(p_a, p_b) = \mathcal{S}(1-p_a, p_a, 1-p_b, p_b, 1-p_a, p_a, 1-p_b, p_b)$ for the supply chain. Stability coefficient p_q decide the failure rate $1-p_q$ and the recovery rate p_q of a warehouse operated by company $q \in \{a, b\}$. Figure 7 shows that the probability of on-time delivery increases when the stability of networks increase.

Figures 8, 9, 10 show the correlation between the stability of networks and the probability of product on-time delivery with three different consolidation strategies: level-2 cross-warehouse consolidation, level-1

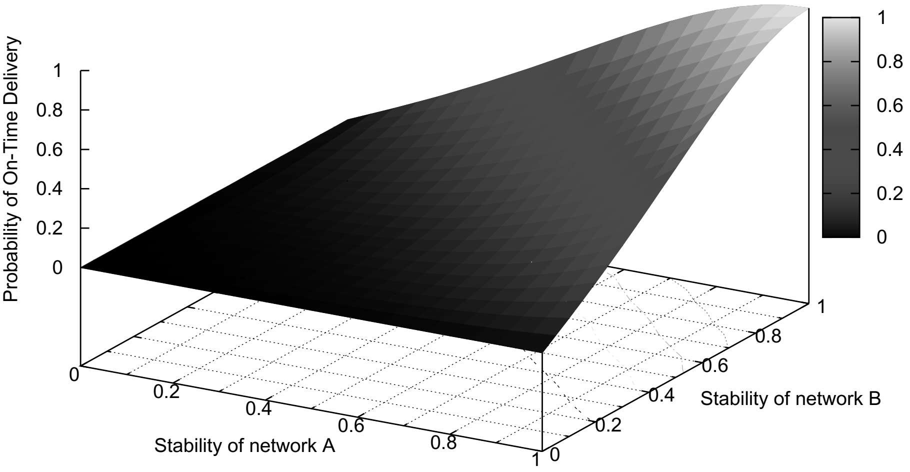


Fig. 8. Stability of networks and their impact on the probability of product on-time delivery with level-1 cross-warehouse consolidation strategy.

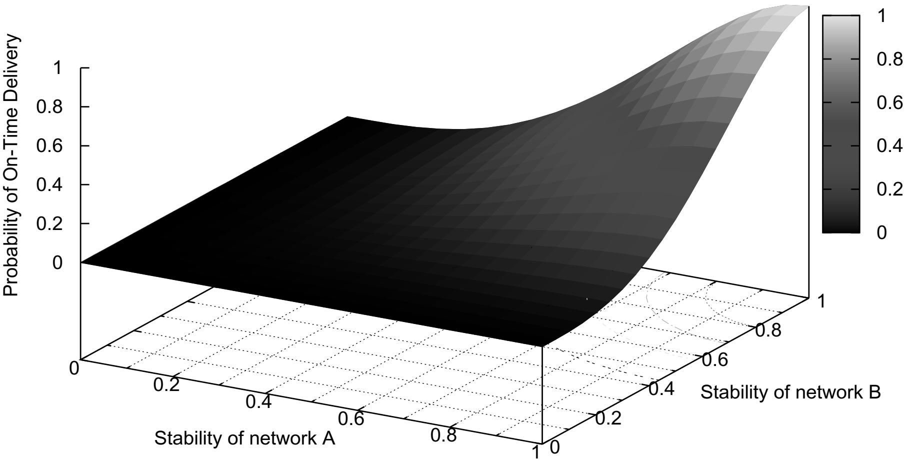


Fig. 9. Stability of networks and their impact on the probability of product on-time delivery with level-2 cross-warehouse consolidation strategy.

cross-warehouse consolidation, and product pooling consolidation. Figure 11 shows the improvement of the probability of on-time delivery for these consolidation strategies. The top surface represents product-pooling consolidation strategy and the bottom surface represents level-2 cross-warehouse consolidation strategy. The improvement is measured by the ratio of the probabilities of on-time delivery after and before a merger. Product pooling consolidation strategy shows a significant improvement on the probability of on-time delivery, level-1 cross-warehouse also shows some moderate improvement, and level-2 cross-warehouse doesn't show any improvement. Our computation study unveils two beneficial factors that help reduce risks in supply chain consolidation,

– *Consolidation taking place closer to suppliers is more beneficial for risk reduction in a hierarchical supply chain.* Figure 12 compare the improvement of possibility of on-time delivery induced by consolidation strategies from the plane $p_a = p_b$, where p_a and p_b are the stability coefficients of networks a and b. It clearly shows that the improvement of the probability of on-time delivery induced by a consolidation strategy increases in the following order: Level-2 cross-warehouse consolidation, Level-1 cross-warehouse consolidation, and product-pooling consolidation. The order also reflects decreased distance from the occurrences of consolidation to suppliers. This is because the consolidation on a high level of a supply chain helps divert risks with little cost in delivery time.

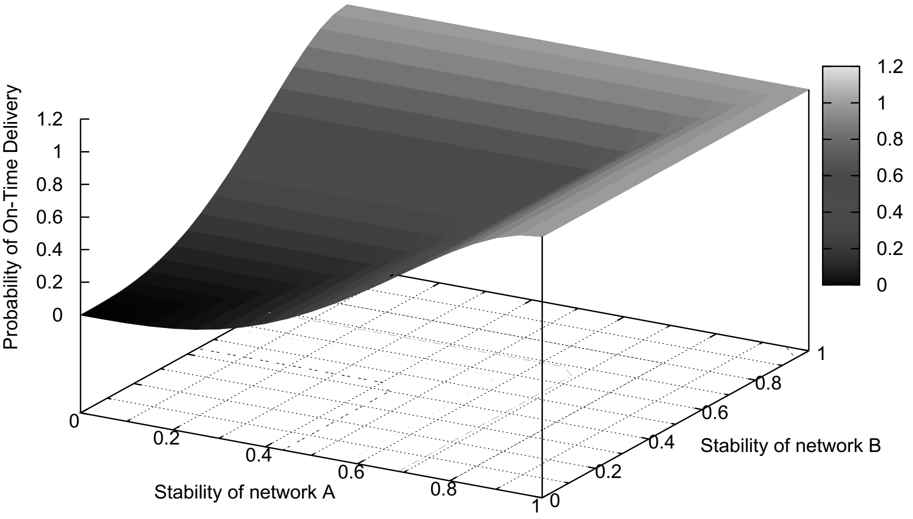


Fig. 10. Stability of networks and their impact on the probability of product on-time delivery with product pooling consolidation strategy.

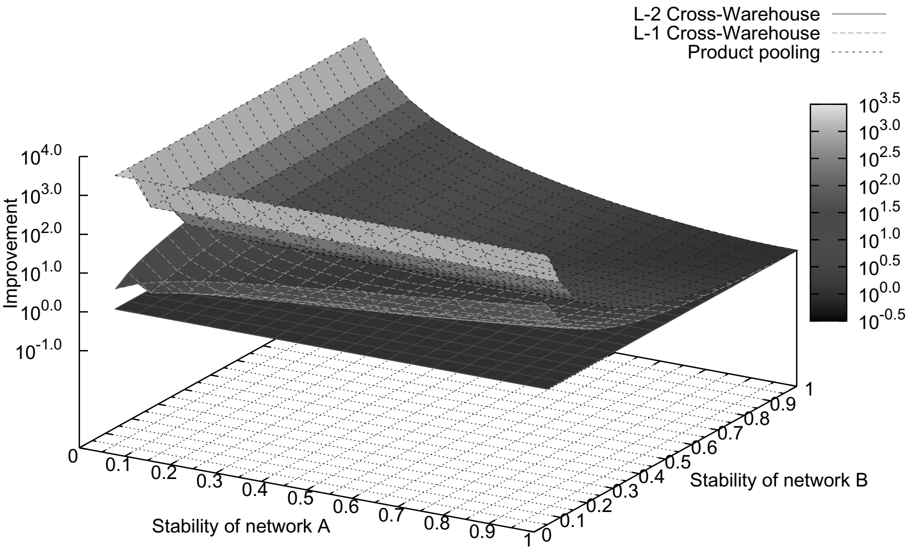


Fig. 11. Improvement of probability of on-time delivery with different supply-chain consolidation strategies.

For instance, in our study, we use the probability of on-time delivery to r_1 and r_2. If company a's network is unreliable, in a consolidated supply chain with product pooling strategy as shown in Fig. 5(d), both s^a and s^b shall send their products to w^b_{11} instead of w^a_{11}. The cost of such diversion is minimal since the delivery time from w^b_{11} shall remain the same as from w^a_{11}. In a consolidated supply chain with level-1 cross-warehouse consolidation strategy as shown in Fig. 5(c), the diversion requires products to be first sent to w^a_{11} and then relayed to w^b_{11}. The cost of such diversion is an extra step on the delivery time so the diversion will eventually reduce the possibility of on-time delivery. Therefore, we still see some benefits of risk reduction in level-1 cross-warehouse strategy, but not as much as in product-pooling strategy. Furthermore, a level-2 cross-warehouse strategy can only help divert risks on the second level of warehouses but not on the first level, and hence its benefit on risk diversion is minimal among all three consolidation strategies in this study.

- *The benefits of consolidations on risk reduction are amplified by unbalanced risks in networks.* Fig-

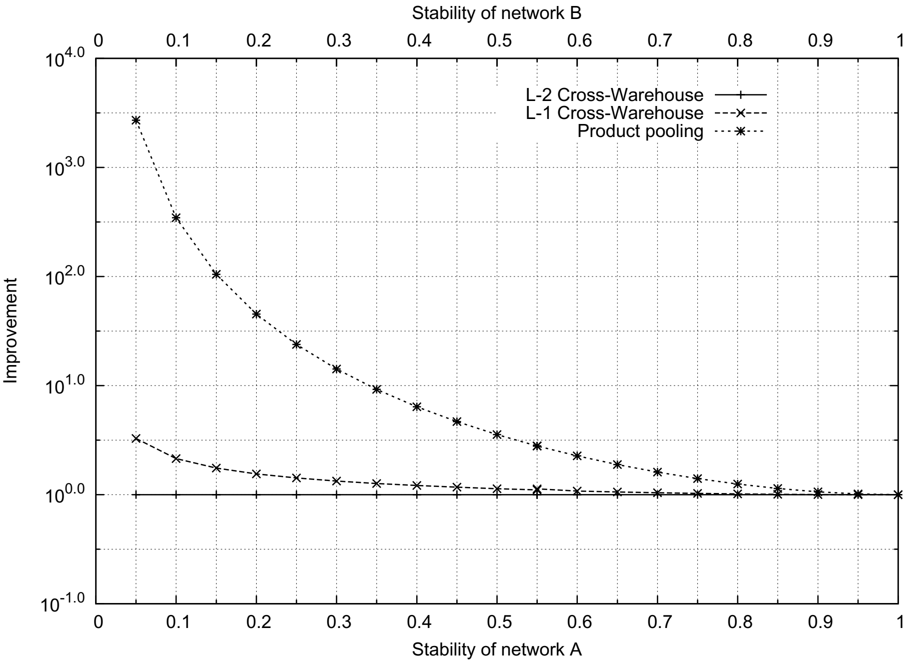


Fig. 12. Improvement of probability of on-time delivery with different supply-chain consolidation strategies where $p_a = p_b$.

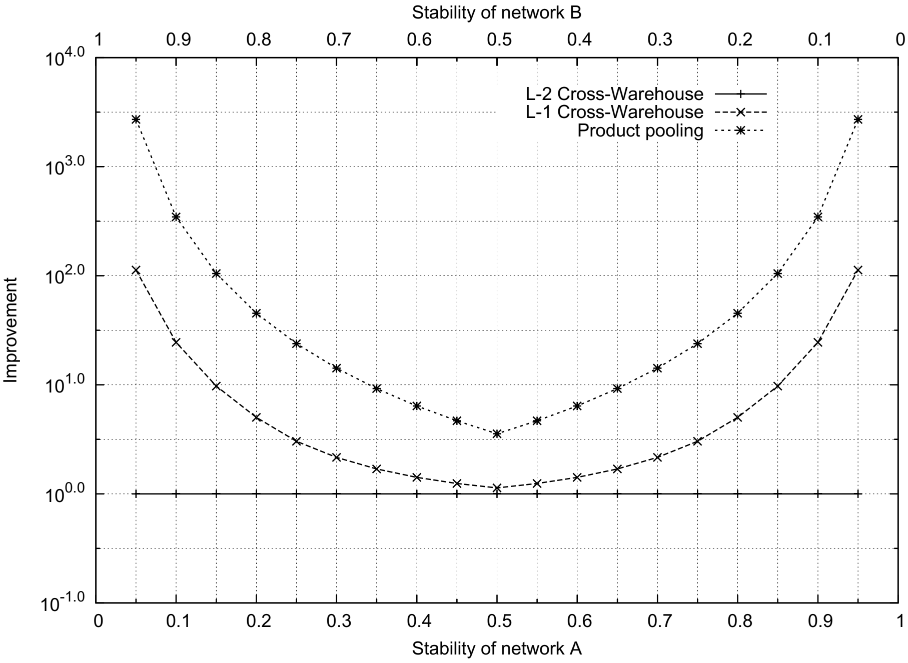


Fig. 13. Improvement of probability of on-time delivery for different supply-chain consolidation strategies where $p_a + p_b = 1$.

ure 13 compares the improvement on the possibility of on-time delivery induced by consolidation strategy from the plane $p_a + p_b = 1$, where p_a and p_b are the stability coefficients of networks a and b. It shows that the consolidation-induced improvement of the probability of on-time delivery is great if risks among individual networks are off balance. An explanation is that, while risks are unevenly distributed among networks, consolidation will help merchandise flow diverse from those high-risk areas. For instance, if most of warehouses in the subnetwork rooting at w^b_{11} fail, the optimal routing algorithm shall choose to dispatch most of merchandise flow, both from companies a and b, through w^a_{11}. If the failure rate of the subnetwork rooting at w^b_{11} is significantly higher than that of the subnetwork rooted at w^a_{11}, such rerouting will greatly improve the overall stability of consolidated networks.

6. Conclusion

In this paper we give a formal evaluation of different supply chain consolidation strategies on their ability of reducing risks in supply chains. To facilitate the computational study, we propose a formal framework for modeling and analyzing stochastic supply chain. The stochastic model of supply chain is based on an extension of Markov Decision Processes. The framework allows the modeling of both nondeterministic behaviors and probabilistic behaviors in one single model. These behaviors reflect dynamics of supply chains. Nondeterministic behaviors represent routing decisions made by supply chain operators, and probabilistic behaviors represent the uncertainty of the operating status of structural elements such as warehouses, routes, etc. To facilitate formal analysis, we translate the objectives of risks analysis into the temporal logic PCTL. We use the probability of on-time product delivery as a barometer of risks in a supply chain. A low degree of the probability of on-time arrival indicates a high degree of risks presenting in a supply chain. We then use an off-the-shelf probabilistic model checker PRISM to analyze risks in supply chains with different consolidation strategies. Unlike traditional methods such as simulation, our model-checking-based approach can compute the possibility of on-time delivery in a given supply chain model by constructing a mathematically sound proof. The soundness comes from model-checking algorithm that use efficient symbolic methods to explore all the possible executions at once, not just one execution like in simulation. Such soundness also improves confidence in risk analysis. Given that, we now can provide rigorous computational study on the comparison of different strategies without having to consider the margin of statistic errors. It shall also note that our approach is complemental to, but not a replacement for, simulation-based approach. Simulation-based approach still enjoys its own advantages. Simulations may be visualized and a simulation-based analysis is in general considered more intuitive. Also, simulation-based approaches in general have a better scalability than formal analysis approaches like ours. This is because simulation-based approach explores only one execution path at a time, whereas our model-checking-based approach checks all the possible execution pathes, although our underlying model checker does so efficiently using symbolic technique [17].

In our computational study we compare three different supply chain consolidation strategies: cross-warehouse shipment consolidation on level-1 warehouses (Level-1 cross-warehouse) and on level-2 warehouses (Level-2 cross-warehouse) as well as product pooling at warehouses from different suppliers (product pooling). The computational study is carried out on a three-echelon supply chain. The results of the computational study have shown two major contributing factors that reduce risks in supply chain consolidation: the first factor is the distance from where consolidation takes place to suppliers. The closer a consolidation takes place to suppliers, the more risk reduction it may introduce; the second factor is the distribution of risks in pre-merger supply chains. Consolidation works better in terms of risk reduction when underlying supply chains previously operated by two companies have unbalanced risks. These findings could help decision makers to adjust their merger strategy when risk is a big concern in their supply chain management. In the future, we would like to evaluate the impact of other factors, such as ordering method and inventory model, on risks in a consolidated supply chain. We also would like to study the evolution of risks during multiple-period supply chain operations. This research introduces a novel risk analysis approach based on probabilistic model checking, which enables us to model and analyze stochastic behaviors of supply chains. In this work, we largely considered routing decisions and their impact on risks in supply chains. As an extension of this work, we will use it to analyze other aspects of supply chain strategies, for example, ordering decisions, and their impact on risks in a supply chain.

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